ɭ ·fnʃtstdslɑ·υjj lsO

tʃts нɑʃfnɣɑʃtʃdzɑʃdɑ «Λ ɑʃʷ tɑtsə·Oʸ tsвtɑ·вtslsʂ·fnʃɑɑdz·dtsə», 6 fnj dzj fnjstɑ

1. ʃ j lstsнɑʃʂ·Otsujdʑɑ

ɹ ·υjj ls tʃ tɑtsətsнɑʃlsfnʸ ə ʷ tstɑdzj tʃɑʃfnj dzj dɑdzsɑts lsj fnjsɑʃtɑtsə·dzʃɑʷ fnɣɑʃfnʃtsdz ɹtsə·dzʃɑj dʒ ɹ dɑ ʂlstɑtsdɑdzs?
ɑʃdzʷ tstɑdʒ·υjɑʃtsdɑdzs-tsвtɑ·Lts·ɔ·lsj dz dzs? fnjɑj нO ɭ ɹ σ υ (http:// www.nstu.ru/sveden/eos).

σj fnjstsəʳ j ɹ ɹ·нɑ·dzʃɑʷ ts⊐ə·Ols ə·Oʸ ls əfnj fnjsнj tɑɣ·dzʃɑj нɑʃfnɣɑʃtʃdzɑʃdz «Λ ɑʃʷ tɑtsə·Oʸ tsвtɑ·вtslsʂ·O fnʃɑɑdz·dtsə».

σj fnjs fnjsfnjstsɑʃls ɑʃ 30 ətstʃtɑtsfnjsə tɑ·Ldzʃɑʃυdzsɑts əɑʃн·O ɑʃ tʃtsɹətsdzʷj ls tʃtɑtsəj tɑɑʃlsᵘ lztɑtsəj dz
fnʸ tstɑdʒ·ʃɑtsə·dzʃɑɑdzsfnjsɑʃ ʂtsdʒʃj lsj dɑɣɑʃʔ ɑʃ fnjstslsdɑfnj dɑdz ⊐ fnɣdzɑʃdʒ·ʃ ɑʃdɑɣɑʃʂ·Olststɑtsə, ɹ·Oʂtɑj tʃ dɑfnj dɑdz ⊐ ɹ·O
нɑʃfnɣɑʃtʃdzɑʃdzs?.

2. υ tɑtsə·dzʃɑʃ tsfnɑtsj dɑʃʷ ʂtsdʒʃj lsj dɑɣɑʃʔ ɑʃ ʂtɑʃlsj tɑɑʃɑʃ tsujdʑɑ

ɹ ʳ tʃtsdzdɑfnjdzɑʃj lsj fnjsO ɹ·Ofnɣɑʃlsᵘ əʊj lsfnʸ dzO **tʃtɑtsнə·ɑʃdzlstsdʒ** lztɑtsə·dɑfnj, j fnɣdzʃ fnjslzнj dzs tʃtɑ·Oəɑʃdz dzs
tslsəj lsɑʃdz dzO əfnj ətstʃtɑtsfnʸ lsj fnjsO, ɹdzʊj ls tstʃtɑj нj dɑfnjdzʃɑʷ əfnj⊐ tʃtsdzʷlsɑʔ, tʃtɑtsнj dʒsdzfnjstɑɑʃtɑtsə·Odz
fnʃtsfnjsвdzsfnjsᵘ вj ɹtsʰ ɑʃвtsυɑʃdzs lzfnjs·Odz·ədzɑʃə·Olsᵘ tʃtsfnɑfnjнtsə·Olsj dz dzsfnjsᵘ ɑʃ fnjstslsəj lsfnjsəɑʃj dʒ ɣ нlz
tʃtɑtsυj fnɣɑʃdʒɑʃ ɑʃ ʷədfnjdzʃɑʷdʒɑʃ, ə ʳ ʷədzʷlsᵘ tʃtɑtsвdfnjdʒ , tʃtɑj нdzO·ɑ·lsᵘ dzj⊐·O·dzʃɑɹ·dʒ ɑʃ⊐ tɑj ʰ j dɑʃʷ,
tʃtɑj нfnjs·ədzʸj ls ʂtsdzʃυj fnjsəj dɑdz j ɑʃ ʂ·Oυj fnjsəj dɑdz j ⊐·Odɑ·lsj tɑɑʃfnjsɑʃʂ·ɑʃ tʃtɑtsυj fnɣfnjsə ɑʃ dfnj нtstʃlzfnjʂ·Oj ls
tsʰ ɑʃвtsʂ· tʃtɑɑʃ tɑj ʰ j dɑʃɑʃɑʃ ɹ·Oн·O υ ʂ tsdʒʃj lsj dɑɣɑʃɑʃ ɑʃ fnjstslsdfnj dɑdz j fnɣ dɑʃɑʃdʒʃ ɑʃdɑɣɑʃʂ·Olststɑʳ ,
ɹ·Oʂtɑj tʃ dfnj dɑdz j ɹ·O нɑʃfnɣɑʃtʃdzɑʃdɑsʔ, fnʸ tstɑdʒ·ʃɑtsə·Odz ə tʃtsdzdɑsdʒ tsвʲ j dzj . ʃ υjj dʑ·O fnjsfnjsədzʸj ls tsls 17
нts 20 в·Odɑsə.

ɹ ʳ tʃtsdzdɑfnjdzɑʃj lsj fnjsO ɹ·Ofnɣɑʃlsᵘ əʊj lsfnʸ dzO **вOɹtsə·tsdʒ** lztɑtsə·dɑfnj, j fnɣdzɑʃ fnjslzнj dzs tʃtɑ·Oəɑʃdz dzs
tslsəj lsɑʃdz dzO l ətstʃtɑtsfnjsə lsj fnjsO, ɹdzʊj ls tstʃtɑj нj dɑfnjdzʃɑʷ tsfnɑfnjsədz ⊐ tʃtsdzʷlsɑʔ, tʃtɑtsнj dʒsdzfnjstɑɑʃtɑtsə·Odz
fnʃtsfnjsвdzsfnjsᵘ lzfnjs·Odz·ədzɑʃə·Olsᵘ tʃtsfnɑfnjнtsə·Olsj dz dzsfnjsᵘ ɑʃ fnjstslsəj lsfnjsəɑʃj dʒ ɣ нlz
tʃtɑtsυj fnɣɑʃdʒɑʃ ɑʃ ʷədfnjdzʃɑʷdʒɑʃ, tʃtɑɑʃ tɑj ʰ j dɑʃɑʃɑʃ ɹ·Oн·O υ нtstʃlzfnjʂ·Oj ls dfnjtʃtɑɑʃɑʃtʃɑʃʂ·Odz dzʔ j tsʰ ɑʃвʂɑʃ,
dzʔ tʃtɑɑʃdʒj tɑ, əʳ υjɑʃfnɑfnjsj dz dzʔ j . ʂ tsdʒʃj lsj dɑɣɑʃɑʃ ɑʃ fnjstslsdfnj dɑdz j fnɣ dɑʃɑʃdʒʃ ɑʃdɑɣɑʃʂ·Olststɑʳ ,
ɹ·Oʂtɑj tʃ dfnj dɑdz j ɹ·O нɑʃfnɣɑʃtʃdzɑʃdɑsʔ, fnjsнj tɑɣ·Os tʃtɑsвj dz ɑʃ fnʸ tstɑdʒ·ʃɑtsə·Odz dzO
вOɹtsətsdʒ lztɑtsə·dfnj . ʃ υjj dʑ·O fnjsfnjsədzʸj ls tsls 14 нts 16 в·Odɑsə.

ɹ ʳ tʃtsdzdɑfnjdzɑʃj lsj fnjsO ɹ·Ofnɣɑʃlsᵘ əʊj lsfnʸ dzO **tʃtɑtsɑtsə·tsdʒ** lztɑtsə·dfnj, j fnɣdzɑʃ fnjslzнj dzs tʃtɑ·Oəɑʃdz dzs
tslsəj lsɑʃdz dzO ½ ï l ətstʃtɑtsfnjsə lsj fnjsO, ɹdzʊj ls tstʃtɑj нj dɑfnjdzʃɑʷ tsfnɑfnjsədz ⊐ tʃtsdzʷlsɑʔ,
tʃtɑtsнj dʒsdzfnjstɑɑʃtɑtsə·Odz fnʃtsfnjsвdzsfnjsᵘ lzfnjs·Odz·ədzɑʃə·Olsᵘ tʃtsfnɑfnjнtsə·Olsj dz dzsfnjsᵘ ɑʃ fnjstslsəj lsfnjsəɑʃj dʒ ɣ нlz
tʃtɑtsυj fnɣɑʃdʒɑʃ ɑʃ ʷədfnjdzʃɑʷdʒɑʃ, tʃtɑɑʃ tɑj ʰ j dɑʃɑʃɑʃ ɹ·Oн·O υ нtstʃlzfnjʂ·Oj ls dfnjtʃtɑɑʃɑʃtʃɑʃʂ·Odz dzʔ j tsʰ ɑʃвʂɑʃ,
dzʔ tʃtɑɑʃdʒj tɑ, əʳ υjɑʃfnɑfnjsj dz dzʔ j . ʂ tsdʒʃj lsj dɑɣɑʃɑʃ ɑʃ fnjstslsdfnj dɑdz j fnɣ dɑʃɑʃdʒʃ ɑʃdɑɣɑʃʂ·Olststɑʳ ,
ɹ·Oʂtɑj tʃ dfnj dɑdz j ɹ·O нɑʃfnɣɑʃtʃdzɑʃdɑsʔ, fnjsнj tɑɣ·Os tʃtɑsвj dz ɑʃ fnʸ tstɑdʒ·ʃɑtsə·Odz dzO tʃtstɑtsɑtsətsdʒ lztɑtsə·dfnj .
ʃ υjj dʑ·O fnjsfnjsədzʸj ls tsls 10 нts 13 в·Odɑsə.

ɹ ʳ tʃtsdzdɑfnjdzɑʃj lsj fnjsO fnɣɑʃlsʂ·Oj lsfnʸ **dfnjlzнtsə·dfnjlsə·tstɑʃlsj dz dzʔ dʒ** j fnɣdzɑʃ fnjslzнj dzs tʃtɑ·Oəɑʃdz dzs
tslsəj lsɑʃdz dzj dfnjj υjj dʒ dzO tʃtsdzts·əɑʃdzʃ ətstʃtɑtsfnjsə lsj fnjsO, dfnj ɹdzʊj ls tstʃtɑj нj dɑfnjdzʃɑʷ? tʃtsdzʷlsɑʔ, dfnj
tʃtɑtsнj dʒsdzfnjstɑɑʃtɑtsə·Odz fnʃtsfnjsвdzsfnjsᵘ lzfnjs·Odz·ədzɑʃə·Olsᵘ tʃtsfnɑfnjнtsə·Olsj dz dzsfnjsᵘ ɑʃ fnjstslsəj lsfnjsəɑʃj dʒ ɣ нlz
tʃtɑtsυj fnɣɑʃdʒɑʃ ɑʃ ʷədfnjdzʃɑʷdʒɑʃ, tʃtɑɑʃ tɑj ʰ j dɑʃɑʃɑʃ ɹ·Oн·O υ нtstʃlzfnjʂ·Oj ls tʃtɑɑʃdɑɣɑʃtʃɑʃʂ·Odz dzʔ j tsʰ ɑʃвʂɑʃ.
ʂ tsdʒʃj lsj dɑɣɑʃɑʃ ɑʃ fnjstslsdfnj dɑdz j fnɣ dɑʃɑʃdʒʃ ɑʃdɑɣɑʃʂ·Olststɑʳ , ɹ·Oʂtɑj tʃ dfnj dɑdz j ɹ·O нɑʃfnɣɑʃtʃdzɑʃdɑsʔ, dfnj
fnʸ tstɑdʒ·ʃɑtsə·Odz . ʃ υjj dʑ·O fnjsfnjsədzʸj ls ***dʒj dfnjj*** 10 ***вOdɑsə***.

3. z¿ Ç¤ΘdzΘtsuj dΔ£ cƒ

ι Θ·uj ls fnμ(cƒlsΘ¿lsfnγ fnμΘdzdz dʒ j¿fnμdzcƒ fnγdcʒkʒΘ·в Θdzdtsə tƒts əfnjdcʒ LΘ·Θdzdcƒʷdʒ вcƒdz¿lsΘ·fnμsfnμsΘədzγj ls tsls 10 нts 20 вΘdzdtsə ə¤dz ιμcƒlsj dz dts. ч lzdcʒkʒΘdcʒj dz¿j 10 вΘdzdtsə tƒtəcƒLdzΘ¿lsfnγ dz¿lzнtsədz¿ls·tstəcƒlsj dz dz dʒ təj Lzdz lsΘstsdʒtƒtətsdcʒj ιlzlstsιμdts? Θkslsj fnμΘuμcƒcƒ tƒts нcƒfnμcƒtƒdzcƒdz¿. вΘdzdz LΘ·LΘujls lzuμcƒlsr əΘγ lsfnγ ə fnμstslsəj lsfnμsəcƒcƒ fnj tƒtəΘəcƒdΘdzʒcƒ вΘdzdz dts-təj?lscƒdzdtsətsə? fnμcƒfnμsj dʒr , lzfnμsΘdtsədz¿dzdz. dʒcƒ ə ι ɹ ʊ υ .

4. ι təcƒdcʒjtəcdz. ?¤ lsj fnμs нdzγ LΘujlsΘ

υ lsəj tə×ιнΘγ :
ιΘə. Ç¤Θwj нtdts? _____ _____ Λ R ∫
«_____» _____ 20_ G.

ʊj fnμs

tƒts нcƒfnμcƒtƒdzcƒdz¿ «Λ cƒwtdtsəΘγ tsвtəΘвtslsÇ¤Θfnμ(GdΔΘdtsə»[1]

1. ι təΘəcƒdts, fnμsΘəwfh j¿j ə fnμstslsəj lsfnμsəcƒj tƒΘtəj ιdz¿dcʒj dzstsə dʒdtsιj fnμsəΘdz¿Ç¤tslststcr ?¤ιdz¿dcʒj dzsιstscΔts ιj¿ dcʒdtsιj fnμsəΘ(dcʒdtsιj fnμsəts **ιΘdz¿Δdzlststs** tslsdtsfnμ(lsj dz dts tstƒj təΘuμcƒ(cƒ)

$\forall a \in M, \forall b \in M: (a, b) \to c \in M$ (часто пишут $a \circ b = c$, обозначая операцию символом $\circ$)

dΔΘLr əΘj lsfnγ

вcƒdΔΘzdts? tstƒj tə·uμcƒj?
Θfnμsuμcƒ·Θscƒədtsfnμsγ ʁ
tƒtəΘəcƒdts fnμzh j¿fnμsətsəΘdz¿cƒʷ dz¿?lstəΘdz dtscΔtsιdz¿dcʒj dzsΘ
lzdcʒdtsιj¿dzcƒj dcʒ

2. ι təcƒdcʒjtəΘdʒcƒ вcƒdΔΘzdz ⊐ tstƒj tə·uμcƒ? ʷədzγʁ lsfnγ:

fnμdtsιj¿dzcƒj¿ uμjdz ⊐ uμ(fnμjdz
lzdcʒdtsιj¿dzcƒj¿ tə·uμ(tsdΔΘdz dz. ⊐ uμ(fnμjdz
нjdzj¿dzcƒj¿ tƒtsdtsιj¿ cƒlsj dz dz. ⊐ əj h j¿fnμsəj¿dzdz. ⊐ uμ(fnμjdz
lzdcʒdtsιj¿dzcƒj¿ əj¿Ç¤lststə·Θ-fnμstscdz uμΘdΔΘəj¿Ç¤lststə-fnμstcts¤Çlz

3. ∫ dtsιj¿fnμsəts M tsвj¿Ç¤lstsə fnμвcƒdΔΘzdts? tstƒj tə·uμcƒj? ⁰, LΘdʒΔdzlstsj tslsdtsfnμlsj dz dtsι lsts? tstƒj tə·uμcƒ(cƒ cƒ lzнtsədz¿lsətstcwʁ h j¿j fnμj¿нlzʁ h cƒdcʒΘ¤fnμ(tscdʒΘdʒ
1) Θfnμsuμcƒ·Θscƒədtsfnμsγ $\forall x, y, z \in M: x \circ (y \circ z) = (x \circ y) \circ z$
2) fnγh j¿fnμsətsəΘdz¿cƒj¿ dz¿?lstəΘdz dtscΔtsι dz¿dcʒj dzsΘe, lsΘ¤tscΔts uμlsts $\forall x \in M: x \circ e = x$
3) fnγh j¿fnμsətsəΘdz¿cƒj¿ нdzγ Ç¤Θ|нtscΔtsι dz¿dcʒj dzsΘtƒtətslsc(ətsƒtscdtsιj¿dtscΔtsj¿dʒlz
$\forall x \in M \; \exists x^{-1} \in M: x \circ x^{-1} = e$
dΔΘLr əΘj lsfnγ

cΔtdztƒtƒts?
dʒtscdtscƒнtscdʒ
tƒtscdz¿dʒ
¤tscdz uμtscdʒ

[1] ι təΘəcƒdz dz. j¿tslsəj lsr ər нj dz¿dz. ×ιcƒtəcdz. dʒh təcƒwlstscdʒ

4. [fgkz[∫ ï ctdzfjfjO¤c , lsts ctdzfjfjO¤dzΩ[r ¤G]lsfmy

§tscbgkbzlsO¤lsc[¤dats?
O¤H¤c[lsc[¤dats?
cbzcdz lsc[fjdz[§tO¤lsc[¤dats?
dz]§tscbgkbzlsO¤lsc[¤dzO¥

5.] tz[cbj tzOb]g[§tscbgkbzlsO¤lsc[¤dz. = ctdzfjfj ¤dz¥ lsfmy:

cbgatsx]j fhjs¤ts ¤ fmj=]uj dz. =]uc[fmjdzfmjtsfjj tzO]uc[j? fgkatsx]j dz[¥
cbgatsx]j fhjs¤ts ¤ fmj= ¤j¡h j fhjs¤j dzdz. =]uc[fmjdzfmjtsfjj tzO]uc[j? fgkatsx]j dz[¥
cbgatsx]j fhjs¤ts ¤fmj= dz]¤r tatsx]H]dzdz. = §¤O¤tzO¤lscdz. = cbG¤lstzc[u]datsctslzcbgatsx]j dz[¥

6. [fgkz[cbgatsx]j fhjs¤ts fmH¤lzcb¥ Bc[dzΩ¤dz. cbg[Gfhfmfsuc[O¤lsc[¤dz. cbg[tsfjj tzO]uc[¥cbg[(tsBr u[dats dzΩ[r ¤G]cb cbg[fgkatsx]j dz[j cb + c[lzcbgatsx]j dz[j cb·), tfjtzc[u[cb tslsdatsfmg[lsj dz dats fgkatsx]j dz[¥ tsdats ¥¤dz¡lsfmy ctdzfjfjts?, O tslsdatsfmg[lsj dz dats lzcbgatsx]j dz[¥ ï tfjtscdzctdzfjfjts?, lsts tsdats dzΩ[r ¤G]lsfmy

ctdzfjfjts?
cbgscdatsc[H¤tscb
tfjtscdz]cb
§tscdz u]tscb

7. [fgkz[§tscdz u]ts fmgH¤j tzx]c[ls¹ cdz]cbj dats, cdz]? lstzO¤dz dz. ? tslsdatsfmg[lsj dz dats lzcbgatsx]j dz[¥, tsdats dzΩ[r ¤G]lsfmy

§tscdz u]tscb fmj H¤c[dz[u]j?
ctdzfjfjts?
tfjtsH¤ctdzfjfjts?
tfjtscdz]cb

8. [fgkz[H¤Odats cbgatsx]j fhjs¤ts tsB¡ j §lstss¤ fmH¤lzcb¥ Bc[dzΩ¤dz. cbg[Gfhfmfsuc[O¤lsc[¤dz. cbg[tsfjj tzO]uc[¥cbg[(tsBr u[dats dzΩ[r ¤G]cb cbg[fgkatsx]j dz[j cb c[lzcbgatsx]j dz[j cb), tfjtzc[u[cb tslsdatsfmg[lsj dz dats fgkatsx]j dz[¥ ¤fmj cbgatsx]j fhjs¤ts ¥¤dz¡lsfmy §tscbgkbzlsO¤lsc[¤dats? ctdzfjfjts?, O tslsdatsfmg[lsj dz dats lzcbgatsx]j dz[¥ §tscbgkbzlsO¤lsc[¤dats? ctdzfjfjts? ¥¤dz¡lsfmy cbgatsx]j fhjs¤ts L O c[ffgdz u]j dz[j cb dzdz¥ (¹ dz]cbj dats O dz]? lstzO¤dz datsctslsdatsfmg[lsj dz dats fgkatsx]j dz[¥), lsts lsO§tsj cbgatsx]j fhjs¤ts dzΩ[r ¤G]lsfmy

dz[dz]?dz. cb tfjtatsfhjstzO¤dzfhjs¤tscb
ctdzfjfjts?
tfjtsH¤ctdzfjfjts?
tfjtscdz]cb

9. § O§c[j tsfjj tzO]uc[c[dz]tsB=tsH¤c[cbgts L O H¤Os dzOcbgatsx]j fhjs¤j tfjtsfmgz]H¤ts¤Oslj dz datsfhjsj?, u[lstsBr tsdats fhjsO¤dats dz[dz]?dz. cb tfjtatsfhjstzO¤dzfhjs¤tscb?

fgkatsx]j dz[j tfjtsfmgz]H¤ts¤Oslj dz datsfhjsj?
lzcbgatsx]j dz[j tfjtsfmgz]H¤ts¤Oslj dz datsfhjs[dzOffgO¤dz¥ tz
lzcbgatsx]j dz[j tfjtsfmgz]H¤ts¤Oslj dz datsfhjsj?
H]dz]cbj dz[j tfjtsfmgz]H¤ts¤Oslj dz datsfhjsj?

10. [fgkz[cbgatsx]j fhjs¤ts M¹ dz]cbj dzatst¤, dzΩ[r ¤G]cb= ¤j§lststzO¤bg[, tsB dzO¤H¤G]ls fmpts?fhjs¤O¤bg[:
    - H¤dz¥ dz Br = ¤j§lststats¤ x c[y c[L M tsfjtz]H]dz]dzO fhjzcbgcbgO tfjtzc[¹ lstscb=tz]Lzdz lsO¤ls tfjtzc[dzO¤H¤dz]x]c[ls M, fgkatsx]j dz[j Gfhfmfsuc[O¤lsc[¤dats c[§tscbgkbzlsO¤lsc[¤dats, ¤ M c[dcbj]lsfmy j H¤c[dzfhjs¤j dzdz. ? dzdz]¤ts?¤j§lststz, O¤H¤dz¥ dz Bts=ts¤j§lststzO x ¤ M fhjz¡ j fhjs¤lz]ls tsBtzO¤lsdz. ? j cbz¤j§lststz;
    - c[dcbj]lsfmy tfjtscdz] F¹ dz]cbj dzatst¤ (fgfgO¤dz¥tats¤), c[tsfjtz]H]dz]dzO tsfjj tzO]uc[¥ lzcbgatsx]j dz[¥ ¤j§lststzO¤dzOffgO¤dz¥tz, tfjtzc[¹ lstscb=lzcbgatsx]j dz[j ¤j§lststzO¤dzOffgO¤dz¥tz Gfhfmfsuc[O¤lsc[¤dats c[H¤c[fhjstzcBlzlsc[¤dats, ¤ tfjtscdz] F c[dcbj¡ lsfmy j H¤c[dz[u]dz. ? c[dzdz]¤ts? fgfgO¤dz¥tz ;

lstsc=H¤OlsO§tsj cbgatsx]j fhjs¤ts ∫ dzΩ[r ¤G]lsfmy é

dz[dz]?dz* tfjtatsfhjstzO¤dzfhjs¤ *

11. [fgkz[cbgatsx]j fhjs¤ts M¹ dz]cbj dzatst¤, dzΩ[r ¤G]cb= ¤j§lststzO¤bg[, tsB dzO¤H¤G]ls fmpts?fhjs¤O¤bg[:

- нdzʸ dz вʳ ⊐ ǝ¡ ʕ¡ lststatsǝ x cʃ y cʃʟ M tstʃtɕ¡ нj dɟ dƶ⊙ fнjzdʒdʒ⊙) tʃtɑʃ ᶤ lstsdʒtɕ¡ ʟzdz lsʘls tʃtɑʃdƶ⊙нdɟ ɤ¡ cʃls M, fнgtsɤ¡ j dƶʃj ⊙fнjtsɤ¡cʃ⊙lscʃǝdtslsʘlscʃǝdts, ǝ M cʃdʒj j lsfʸ j нcʃdzfнjǝ¡ dƶdz ʔ dƶzdɟ ǝtsʔ ǝ¡ ʕ¡ lststc, ⊙нdzʸ dz вtsɑts ǝ¡ ʕ¡ lststɑ⊙x ǝ M fнjzʰ j fнjǝ¡ lz¡ ls tsвtɑ⊙lsdz ʔ j dʒz ǝ¡ ʕ¡ lststc;
lstsɑ⊙нls⊙ʕtsj dʒdtsɤ¡ j fнjǝts tsвtɑ⊙Lz¡ ls é

ʕtsdʒdʒzls⊙ls* ɑtdztʃtʃ*
⊙вj dɟ ǝ* ɑtdztʃtʃ*

12. ʗ fngtzʃ tstʃtɕ¡ нj dɟ dz tstʃ j tɑ⊙ɣcʃcʃ ǝ¡ ʕ¡ lststɑtdtsɑts fngtsɤ¡ j dƶ[w cʃ lzdʒdtsɤ¡ j dƶ[w ǝ¡ ʕ¡ lststɑ⊙dƶ⊙fnʕ⊙dzʸtɑ, lsts tstʃtɕ¡ нj dɟ dƶ⊙ ʕtsdɟ ɤcdƶ⊙ fnɟzdʒdʒ⊙ ∑ⁿᵢ₌₁ ₓᵢ ₓ̄ нdzʸ dz вtsʔ fnjǝtsʕ lztʃ dtsfnjscʃ K ǝ¡ ʕ¡ lststatsǝ ₓ̄, ₓ̄ ∈ 1, ₓ̄ fnjs fnʕ⊙dzʸtɑdz dʒcʃ ʕtsᵘ ʷ ʷ cʃɣcʃj dtsʘdʒcʃ ₓ̄ ∈ ₓ̄, ₓ̄ ∈ 1, ₓ̄, cʃ ᶤ ls⊙fnjzdʒdʒ⊙dƶ⊙Lʳ ǝ⊙j lsfʸ é

dƶcʃdɟj ʔ dzʸ ʕtsdʒǝcʃdƶ⊙ɣ[*

13. ʗ fngtzʃ ₓ̄ ∈ ∑ⁿᵢ₌₁ ₓᵢ ₓ̄=0 lstsɑнs⊙cʃ lstsdz ʕts lstsɑнs⊙) ʕtsɑнs⊙ₓ̄ ∈ 0 ∀ₓ̄, lsts ls⊙ʕ⊙ʸ fnjǝtsʕlztʃ dtsfnjs ǝ¡ ʕ¡ lststatsǝ ₓ̄ₓ̄, ₓ̄ ∈ 1, ₓ̄ₓ̄ dƶ⊙Lʳ ǝ⊙j lsfʸ é

dƶcʃdɟj ʔ dzʸ dɟ ʟ⊙ǝ cʃfngʃcdʒ*

14. ʉ tsǝtsʕlztʃ dtsfnjsʲ dƶcʃdɟj ʔ dtsdɟ ʟ⊙ǝcʃfngʃcdʒ ⊐ ǝ¡ ʕ¡ lststatsǝ, lsʕʕ⊙ʸ, ɣ¡sts dz вtsʔ ǝ¡ ʕ¡ lststɑcʃ ʟ нʘdzdtsɑts tʃtɑtsfnjstɑʘdzfnjsǝ⊙ dʒsɤ¡ dtsdtʃtɕ¡ нfnjs⊙ǝcʃlsʲ dƶcʃdɟj ʔ dtsʔ ʕtsdʒǝcʃdƶ⊙ɣcʃcʃj ʔ нʘdzdtsʔ fnjǝtsʕlztʃ dtsfnjscʃ, dƶ⊙Lʳ ǝ⊙j lsfʸ é

вʘLɑcʃfnʃ

15. ʗ fngtzʃ ʟG¥ cʃʕfngʃtatsǝ⊙ls ʲ ǝ¡ ʕ¡ lststɑᶜ ₓ̄, ₓ̄ ∈ 1, ₓ̄, lsts tʃtsdzɣu⊙j dʒ j tʃtɑʃ ǝfnjǝtsʟdʒtsɤ¡ dz ⊐ dƶ⊙вtstɑ⊙ʕ ₓ̄ₓ̄ ∈ ₓ̄, ₓ̄ ∈ 1, ₓ̄ₓ̄ dƶcʃdɟj ʔ dz j ʕtsdʒǝcʃdƶ⊙ɣcʃcʃ tsвtɑ⊙Lzᵛ ls é нʘdzdtsʔ fnjǝtsʕlztʃ dtsfnjscʃ ǝ¡ ʕ¡ lststatsǝ.

dƶcʃdɟj ʔ dzʸ tsвtsdtsɣ[*

16. ∫ cʃdɟj ʔ dƶ⊙ʸ tsвtsdtsɣ[⊙fnjǝtsʕlztʃ dtsfnjscʃ dƶcʃdɟj ʔ dtsdɟ ʟ⊙ǝcʃfngʃcdʒ ⊐ ǝ¡ ʕ¡ lststatsǝ ₓ̄ₓ̄, ₓ̄ ∈ 1, ₓ̄ₓ̄ tʃtɕ¡ нfnjs⊙ǝdzʸj ls fnjsвtsʔ tʃtɑtsfnjstɑʘdzfnjsǝts, lsts нdzʸ нʘdzdtsɑts tʃtɑtsfnjstɑʘdzfnjsǝ⊙dƶ⊙вtstɑ⊙ǝ¡ ʕ¡ lststatsǝ ʷǝdzʸj lsfʸ é

вʘLɑcʃfnʃ

17. ∫ cʃdɟj ʔ dƶ⊙ʸ tsвtsdtsɣ[⊙fnjǝtsʕlztʃ dtsfnjscʃ dƶcʃdɟj ʔ dtsdɟ ʟ⊙ǝcʃfngʃcdʒ ⊐ ǝ¡ ʕ¡ lststatsǝ ₓ̄ₓ̄, ₓ̄ ∈ 1, ₓ̄ₓ̄ tʃtɕ¡ нfnjs⊙ǝdzʸj ls fnjsвtsʔ tʃtɑtsfnjstɑʘdzfnjsǝts, ɑнj ₓ̄ ᶤ lstsé

tɑʘLɑdʒj tɑ*
tɑʘLɑdʒj tɑ* tʃtɑtsfnjstɑʘdzfnjsǝ *

18. ʗ dzʸ dz вtsɑts tʃtɑtsfnjstɑʘdzfnjsǝ⊙вʘLɑcʃfnɟdʒsɤ¡ dtsʟ⊙н⊙ls ʲ

dʒdtsɤ¡ j fnjs⊙ tsdʒ fngʃtsfnjsвtsǝ
j нcʃdzfnjsǝj dƶdz dʒtsвtɑ⊙Ltsdʒ
tsɑtɑ⊙dzʃuj dƶdz dʒʕtsdƶʃuj fnjs⊙tsdʒ fngʃtsfnjsвtsǝ

19. ʗ fngtzʃ dʒdtsɤ¡ j fnjs⊙ts ǝfnj⊐ dzcʃdɟj ʔ dz ⊐ ʕtsdʒǝcʃdƶ⊙ɣcʃcʃ ʔ ǝcʃн⊙ ∑ⁿᵢ₌₁ ₓᵢ ₓ̄ₓ̄ ($\{x_k, k = \overline{1,K}\}$ fnjsǝtsʕlztʃ dtsfnjs ʲ dzcʃdɟj ʔ dtsdɟ ʟ⊙ǝcʃfngʃcdʒ ⊐ ǝ¡ ʕ¡ lststatsǝ) tʃtɑʃ ǝfnj⊐ǝtsʟdʒtsɤ¡ dz ⊐ dƶ⊙вtstɑ⊙ʕ ǝ¡ fnjsǝʳ ⊐ ʕtsᵘ ʷ ʷ cʃɣcʃj dtsstsǝ ₓ̄ₓ̄ fnjsǝtʃ⊙нGj ls fnj tʃtɑtsfnjstɑʘdzfnjsǝtsdʒ M, lsts ls⊙ʕtsʔ вʘLɑcʃfnɟdƶ⊙Lʳ ǝ⊙j lsfʸ é

tʃtsdtdz *

20. ʏ lsts ɑ⊙tɑ⊙dtsʃtɑdzj ls ǝtsʟdʒtsɤ¡ dtsfnjs ʲ tʃtɕ¡ нfnjs⊙dɟ dɑ[w ǝ н⊙dzdtsdʒвʘLɑcʃfnɟ dz вtsɑts fngʃɑdƶ⊙dz⊙cʃʟ н⊙dzdtsɑts tʃtɑtsfnjstɑʘdzfnjsǝ⊙?

tʃtsdtdtsls⊙
tʃtsdtdz ʔ вʘLɑcʃfnɟ
вʘLɑcʃfnʃ tʃtsdtdz *

21. ɹ tɑtsfnjstɑʘdzfnjsǝts fnjs fnʕ⊙dzʸtɑdz dʒtʃtɑtscʃlǝj ¡ нj dɑ[j dʒ cʃ tʃ tstɑtsɤ¡ нj dtdtsʔ cʃdʒ dtstɑdʒɤs? cʃ dʒj lstɑ[ʕtsʔ, tsвdƶ⊙нʘʸ ʰ j¡ j fnjɣtsʔ fnjǝtsdʒtʃtsdtdtsls ʳ , dƶ⊙Lʳ ǝ⊙j lsfʸ é

ɑ̈ʃdz ʁjtdstsɔ*

22. ʃ tʃjtɑʘuɳʃʷ, ʟʘʜʘɑzdzʘʸ ʜdzʷ dz ʁʳ ⊐ ʜɔlz⊐ tʃtsfɳɡjʜtsɔʘsjdz dɑsfɳsjʔ x[n] ɖ y[n] ɔʳ tɑʘɤjdɑʃjdʒ, =
∑ *[], dɑʘʟʳ ɔGjlsfɳʸ é

fɳʃʘdzʷtɑdz* tʃtɑtsʃɪɔjʜ*
fɳʃʘdzʷtɑdz* lzdʒdɑtsɤjdɑʃ*
ɔ dzlstɑjdɑdz* tʃtɑtsʃɪɔjʜjdz*

23. ʌ ɖɪɑʃujfɳɖʔ fɳɡʒʳ fɳɡzdɑstɑdʒʳ ʜdzʷ tʃtɑtsfɳstɑʘdzfɳsɔʘℓ2 fɳtsfɳstsʃlsɔ lstsdʒ ɯlstsé

dɑstɑdʒʘtɑʘɔ dɑʘʕɜɔʘɳtɑʘlsdɑtsdʒzʕtstɑdzɜ ɖɪɑʳdɑjtɑɑʃɖʃ tʃtsfɳɡjʜtsɔʘsjdz dɑsfɳsɖʃ;
ɔɔjʜjdɑɖjdʒdɑstɑdʒʳ ɪʘʜʘʸ ls tʃtsfɳɡjʜtsɔʘsjdz dɑsfɳsɖʃ tsɑʔtʘdɑʃɯjdɑdɑsʔ ʳdjtɑɑʃɖʃ, ʕtslstsʈɑʳj tsʁtɑʘʟzʷ ls ʁjfɳʕtsdjɯdɑstdjtɑdɑsj tʃtɑtsfɳstɑʘdɑfɳsɔtʂ;
dɑstɑdʒʘtstʃtɑjʜjdzʷjls djtsʁ⊐tsʜɖʃdʒtsj tɑʘfɳɡjstsʷdɑɖj dʒjɤʜlzɔjʕlstsʈɑʘdʒʃ;

24. ɻ tɑtsfɳstɑʘdɑfɳsɔtʂ, ɔ ʕtslstsʈɑtsdʒɔɔjʜjdɑʘdɑstɑdʒʘ dɑʘʟʳ ɔGjlsfɳʸé

dɑstɑdʒʃtɑtsɔ Θdz*

25. ɻ tɑtsfɳstɑʘdɑfɳsɔtʂ, ɔ ʕtslstsʈɑtsdʒtstʃtɑjʜjdjdɑʘdʒjlstɑʃʕʘ dɑʘʟʳ ɔGjlsfɳʸé

dʒjlstɑʃujfɳɖ*

26. ʃ ɖdjʔdɑtsj tʃtɑtsfɳstɑʘdɑfɳsɔtʂ , ɖdʒjjls dɑstɑdʒz

$$\|\|_{} = \sqrt{\quad |\|^{}}$$

$$\|\|_{} = \quad |\|$$

$$\|\|_{} = \sqrt{\quad |\|^{}}$$

27. ɾ fɳɡzʃ fɳʃʘdzʷtɑdɑsj tʃtɑtsɖɪɔjʜjdɑʃj ʜɔlz⊐ tʃtsfɳɡjʜtsɔʘsjdz dɑsfɳsjʔ tɑʘɔdɑs dɑzdz , lstslsʘʕɖjtʃtsfɳɡjʜtsɔʘsjdz dɑsfɳsɖʃ ʷɔdzʷʁ lsfɳʸ é

tstdstsɖʔtsdɑʘdz dz*

28. ɾ fɳɡzʃ ʜɔjtʃtsfɳɡjʜtsɔʘsjdz dɑsfɳsɖʃ tstdstsɖʔtsdɑʘdz dz , lstsɖ⊐ fɳʃʘdzʷtɑdɑsjtʃtɑtsɖɪɔjʜjdɑʃjtɑʘɔdɑs é
dɑzdz*
0
dɑstdz*

29. ɻ tɑtsfɳstɑʘdɑfɳsɔtʂ tʃtsfɳɡjʜtsɔʘsjdz dɑsfɳsjʔ fɳtsfɳʃʘdzʷtɑdz dʒtʃtɑtsɖɪɔjʜjdɑʃjdʒɖtʃtstɑtsɤjʜjdɑdɑsʔ ɖdʒdɑstɑdʒtsʔ
$\|\|_{} = \sqrt{\sum \quad |\|^{}}$ ʷɔdzʷjlsfɳʸ

tʃtsdɑdz dʒ
ɑ̈ʃdz ʁjtdstsɔʳ dʒ
dɑstɑdʒʃtɑtsɔ Θdɑdz dʒ
tɑʘɔdɑsdjtɑdz dʒ
tstdstsɖʔtsdɑʘdz dz dʒ

30. ʃ jlstɑʃʕʘɪʟʘʜɖlsé dʒjɤʜlz ʜɔlzdʒʷ tʃtsfɳɡjʜtsɔʘsjdz dɑsfɳsʷdɑʃ.

tɑʘɖfɳɡjstsʷ dɑʃj